On the electromagnetic nature of dark energy and the origin of cosmic magnetic fields

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In this work we consider quantum electromagnetic fields in an expanding universe. We start by reviewing the difficulties found when trying to impose the Lorenz condition in a time-dependent geometry. Motivated by this fact, we explore the possibility of extending the electromagnetic theory by allowing the scalar state which is usually eliminated by means of the Lorenz condition to propagate, preserving at the same time the dynamics of ordinary transverse photons. We show that the new state cannot be generated by charged currents, but it breaks conformal invariance and can be excited gravitationally. In fact, primordial quantum fluctuations produced during inflation can give rise to super-Hubble temporal electromagnetic modes whose energy density behaves as a cosmological constant. The value of the effective cosmological constant is shown to agree with observations provided inflation took place at the electroweak scale. The theory is compatible with all the local gravity tests and is free from classical or quantum instabilities. Thus we see that, not only the true nature of dark energy can be established without resorting to new physics, but also the value of the cosmological constant finds a natural explanation in the context of standard inflationary cosmology. On sub-Hubble scales, the new state generates an effective charge density which, due to the high electric conductivity of the cosmic plasma after inflation, gives rise to both vorticity and magnetic fields. Present upper limits on vorticity coming from CMB anisotropies are translated into lower limits on the present value of cosmic magnetic fields. We find that magnetic fields $B_\lambda > 10^{-12}$ G can be typically generated with coherence lengths ranging from sub-galactic scales up to the present Hubble radius. Those fields could act as seeds for a galactic dynamo or even account for observations just by collapse and differential rotation of the protogalactic cloud.

§1. Introduction

Maxwell's electromagnetism and its quantum counterpart, Quantum Electrodynamics (QED), provide a very accurate description of electromagnetic propagation and interactions in a huge range of scales, from the extremely small distances probed by high-energy colliders and cosmic rays, up to the coherence lengths around 1.3 A.U. of the magnetic fields dragged by the solar wind.[1)] However, the behaviour of electromagnetic fields with wavelengths larger than the solar system radius is still far from clear. Indeed, not only we do not have experimental access to such a low energy regime, but more importantly, the standard theory does not seem to be able to explain the origin of the cosmic magnetic fields found in galaxies, clusters[2)] and, very recently,[3)] also in the voids.

A similar situation is found for gravity, the other long-range interaction in nature.

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Although General Relativity (GR) provides a good description of gravity from sub-millimeter scales up to solar system distances, several problems on larger scales have led to postulate the existence of exotic components in the universe, i.e. dark matter and dark energy, or even to consider modifications of GR on cosmological scales.

The above discussion suggests that a more careful analysis of the behaviour of electromagnetic fields in cosmological contexts is needed. A fundamental aspect of this problem which has not received much attention is the issue of quantization of gauge theories in time-dependent geometries. As is well-known, due to the redundancy in the description of gauge fields, all the quantization approaches restrict in some way the possible physical states of the theory. Thus, for example, most of the studies performed in expanding backgrounds have been based in the use of the Coulomb gauge fixing condition. In flat space-time, Coulomb quantization is equivalent to other methods such as covariant (Gupta-Bleuler) or BRST quantization. However, in a curved space-time this equivalence has not been proved. As a matter of fact, Gupta-Bleuler formalism has been shown to be ill-defined in a time-dependent background,[4),5)] and also BRST method has been shown to exhibit similar pathologies in certain space-time geometries.[6)] These difficulties are rooted in the impossibility of uniquely decomposing the fields in their positive and negative frequency modes in a curved space-time. This fact, in turn, prevents a unique definition of the space of physical states in those formalisms.

In this work, we explore an extended theory of electromagnetism which avoids the above mentioned problems by allowing the propagation of the state which is usually eliminated by means of the subsidiary condition. The price to pay is the modification of Maxwell's equations. However, this modification does not affect the dynamics of ordinary transverse photons and, what is more remarkable, it introduces an effective current in the equations which, unlike ordinary equations, allow the generation of cosmic magnetic fields all the way from sub-galactic scales up to the present Hubble radius.[7)] On the other hand, on super-Hubble scales, it can be seen that the new state contributes as an effective cosmological constant.[8)] Interestingly, because of the breaking of conformal triviality in the modified theory, the new state can be excited from quantum fluctuations during inflation. Thus, in a completely analogous way to the generation of metric and density perturbations, in the extended theory, an almost scale invariant spectrum of electromagnetic perturbations is also produced. The correct amplitude required to explain the present phase of accelerated expansion of the universe and the observed cosmic magnetic fields can be obtained in a natural way provided inflation took place at the electroweak scale.

§2. Covariant quantization in flat space-time

Let us start by briefly reviewing the standard covariant quantization method in Minkowski space-time[9)] since this will be useful in the rest of the work. The starting point is the modified electromagnetic action:

$$S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\partial_\mu A^\mu)^2 + A_\mu J^\mu \right). \qquad (2\cdot1)$$

Because of the presence of the gauge breaking ξ-term, this action is no longer invariant under arbitrary gauge transformations, but only under residual ones given by: $A_\mu \to A_\mu + \partial_\mu \theta$, provided $\Box\theta = 0$. The equations of motion obtained from this action now read:

$$\partial_\nu F^{\mu\nu} + \xi \partial^\mu (\partial_\nu A^\nu) = J^\mu. \qquad (2\cdot2)$$

In order to recover ordinary Maxwell's equation, the Lorenz condition $\partial_\mu A^\mu = 0$ must be imposed so that the ξ-term disappears. At the classical level this can be achieved by means of appropriate boundary conditions on the field. Indeed, taking the four-divergence of the above equation, we find:

$$\Box(\partial_\nu A^\nu) = 0 \qquad (2\cdot3)$$

where we have made use of current conservation. This means that the field $\partial_\nu A^\nu$ evolves as a free scalar field, so that if it vanishes for large $|t|$, it will vanish at all times. At the quantum level, the Lorenz condition cannot be imposed as an operator identity, but only in the weak sense $\partial_\nu A^{\nu\,(+)}|\phi\rangle = 0$, where $(+)$ denotes the positive frequency part of the operator and $|\phi\rangle$ is a physical state. This condition is equivalent to imposing $[\mathbf{a}_0(\vec{k}) + \mathbf{a}_\parallel(\vec{k})]|\phi\rangle = 0$, with $\mathbf{a}_0$ and $\mathbf{a}_\parallel$ the annihilation operators corresponding to temporal and longitudinal electromagnetic states. Thus, in the covariant formalism, the physical states contain the same number of temporal and longitudinal photons, so that their energy densities, having opposite signs, cancel each other. Therefore, only the transverse photons contribute to the energy density.

§3.   Covariant quantization in an expanding universe

Let us consider the curved space-time version of action $(2\cdot1)$:

$$S = \int d^4 x \sqrt{g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2} (\nabla_\mu A^\mu)^2 + A_\mu J^\mu \right]. \qquad (3\cdot1)$$

Now the modified Maxwell's equations read:

$$\nabla_\nu F^{\mu\nu} + \xi \nabla^\mu (\nabla_\nu A^\nu) = J^\mu \qquad (3\cdot2)$$

and taking again the four divergence, we get:

$$\Box(\nabla_\nu A^\nu) = 0. \qquad (3\cdot3)$$

We see that once again $\nabla_\nu A^\nu$ behaves as a scalar field which is decoupled from the conserved electromagnetic currents, but it is non-conformally coupled to gravity. This means that, unlike the flat space-time case, this field can be excited from quantum vacuum fluctuations by the expanding background and this poses the question of the validity of the Lorenz condition at all times. Thus, for example, let us consider as a toy example the quantization in an expanding background interpolating between two asymptotically flat regions: $ds^2 = a(\eta)^2(d\eta^2 - d\vec{x}^2)$ with $a(\eta) = 2 + \tanh(\eta/\eta_0)$ where η_0 is constant.
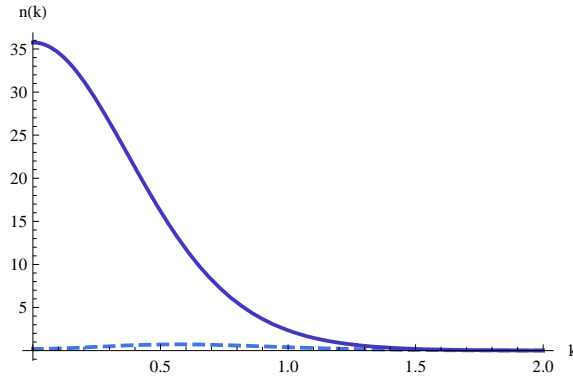
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Fig. 1. Occupation numbers for temporal (continuous line) and longitudinal (dashed line) photons in the *out* region vs. k in η_0^{-1} units.

If we prepare our system in an initial state $|\phi\rangle$ belonging to the physical Hilbert space, for example with $n_0^{out}(k) = n_\parallel^{out}(k) = 0$, i.e. satisfying $\partial_\nu \mathcal{A}_{in}^{\nu\,(+)}|\phi\rangle = 0$ in the initial flat region. Because of the expansion of the universe, the positive frequency modes in the *in* region with a given temporal or longitudinal polarization will become a linear superposition of positive and negative frequency modes in the *out* region and with different polarizations.[4] Thus, the system will end up in a final state which no longer satisfies the weak Lorenz condition i.e. in the *out* region $n_0^{out}(k) \neq n_\parallel^{out}(k)$ or in other words, $\partial_\nu \mathcal{A}_{out}^{\nu\,(+)}|\phi\rangle \neq 0$ as shown in Fig. 1.

A similar problem with the subsidiary conditions has been recently found in BRST quantization in Rindler space-time.[6] Thus, in this work we will explore the possibility of quantization in an expanding universe without imposing the Lorenz condition.

§4. Extended electromagnetism without the Lorenz condition

Let us then explore the possibility that the fundamental theory of electromagentism is given by the modified action (3·1) where we allow the $\nabla_\mu A^\mu$ field to propagate. Since we are not imposing the Lorenz condition, in principle, important viability problems for the theory could arise, namely: modification of classical Maxwell's equations, new unobserved photon polarizations, negative norm (energy) states or conflicts with QED phenomenology. However, as we will show in the following, none of these problems is actually present.

Having removed one constraint, the theory contains one additional degree of freedom. Thus, the general solution for the modified equations (3·2) can be written as:

$$\mathcal{A}_\mu = \mathcal{A}_\mu^{(1)} + \mathcal{A}_\mu^{(2)} + \mathcal{A}_\mu^{(s)} + \partial_\mu \theta \qquad (4\cdot1)$$

where $\mathcal{A}_\mu^{(i)}$ with $i = 1, 2$ are the two transverse modes of the massless photon, $\mathcal{A}_\mu^{(s)}$ is the new scalar state, which is the mode that would have been eliminated if we had imposed the Lorenz condition and, finally, $\partial_\mu \theta$ is a purely residual gauge mode,

which can be eliminated by means of a residual gauge transformation in the asymptotically free regions, in a completely analogous way to the elimination of the A_0 component in the Coulomb or Lorenz gauge quantization. The fact that Maxwell's electromagnetism could contain an additional scalar mode decoupled from electromagnetic currents, but with non-vanishing gravitational interactions, was already noticed in a different context in.[10]

In order to quantize the free theory, we perform the mode expansion of the field with the corresponding creation and annihilation operators for the *three* physical states:

$$\mathcal{A}_\mu = \int d^3\vec{k} \sum_{\lambda=1,2,s} \left[\mathbf{a}_\lambda(k)\mathcal{A}_{\mu k}^{(\lambda)} + \mathbf{a}_\lambda^\dagger(k)\overline{\mathcal{A}_{\mu k}^{(\lambda)}} \right] \tag{4·2}$$

where the modes are required to be orthonormal with respect to the appropriate scalar product. Notice that the three modes can be chosen to have positive normalization, and therefore:

$$\left[\mathbf{a}_\lambda(\vec{k}), \mathbf{a}_{\lambda'}^\dagger(\vec{k'}) \right] = \delta_{\lambda\lambda'}\delta^{(3)}(\vec{k} - \vec{k'}), \quad \lambda, \lambda' = 1, 2, s \tag{4·3}$$

We see that the sign of the commutators is positive for the three physical states, i.e. the negative norm state can be eliminated in the free theory.

The evolution of the new mode is given by (3·3), so that on super-Hubble scales $(|k\eta| \ll 1)$, $|\nabla_\mu \mathcal{A}_k^{(s)\mu}| = const.$ which, as shown in,[8] implies that the field contributes as a cosmological constant in (3·1). Indeed, the energy-momentum tensor derived from (3·1) reads in that limit:

$$T_{\mu\nu} = \frac{\xi}{2}g_{\mu\nu}(\nabla_\alpha A^\alpha)^2 \tag{4·4}$$

which is the energy-momentum tensor of a cosmological constant. Notice that, as seen in (3·3), the new scalar mode is a massless free field and it is possible to calculate the corresponding power spectrum generated during inflation, $P_{\nabla A}(k) = 4\pi k^3 |\nabla_\mu \mathcal{A}_k^{(s)\mu}|^2$. In the super-Hubble limit, we get in a quasi-de Sitter inflationary phase characterized by a slow-roll parameter ϵ:

$$P_{\nabla A}(k) = \frac{9H_{k_0}^4}{16\pi^2}\left(\frac{k}{k_0}\right)^{-4\epsilon} \tag{4·5}$$

where H_{k_0} is the Hubble parameter when the k_0 mode left the horizon and we have chosen the normalization so that $\xi = 1/3$ (see[8]). Notice that this result implies that $\rho_A \sim \frac{9H_{k_0}^4}{64\pi^2\epsilon}\left(\frac{H_0}{k_c}\right)^{4\epsilon}$. Taking the infrared cutoff k_c as the comoving Hubble radius at the beginning of inflation (see[11] and references therein for problems with infrared divergences during inflation), the measured value of the cosmological constant then typically requires $H_{k0} \simeq 2 \times 10^{-6}$ eV, which corresponds to an inflationary scale $M_I \sim 100$ GeV. Thus we see that the cosmological constant scale can be naturally explained in terms of physics at the electroweak scale. This is one of the most

relevant aspects of the present model in which, unlike existing dark energy theories based on scalar fields, dark energy can be generated without including any potential term or dimensional constant.

As shown above, the field amplitude remains frozen on super-Hubble scales, so that no modification of Maxwell's equation is generated on those scales, however as the amplitude starts decaying once the mode enters the horizon in the radiation or matter eras, the ξ term in (3·2) generates an effective current which can produce magnetic fields on cosmological scales, as we will show below.

Notice that in Minkowski space-time, the theory (3·1) is completely equivalent to standard QED. This is so because, although non-gauge invariant, the corresponding effective action is equivalent to the standard BRST invariant effective action of QED.[4]

To summarize, none of the above mentioned consistency problems for the theory in (3·1) arise, thus: the new state can only be generated gravitationally and evades laboratory detection; the new state has positive norm (energy); the effective action is completely equivalent to standard QED in the flat space-time limit and although ordinary Maxwell's equations are modified on small scales, the only effect of the new term is the generation of cosmic magnetic fields.

On the other hand, despite the fact that the background evolution in the present case is the same as in ΛCDM, the evolution of metric perturbations could be different. We have calculated the evolution of metric, matter density and electromagnetic perturbations.[12] The propagation speeds of scalar, vector and tensor perturbations are found to be real and equal to the speed of light, so that the theory is classically stable. On the other hand, it is possible to see that all the parametrized post-Newtonian (PPN) parameters[13] agree with those of General Relativity, i.e. the theory is compatible with all the local gravity constraints for any value of the homogeneous background electromagnetic field.[8],[14]

Concerning the evolution of scalar perturbations, we find that the only relevant deviations with respect to ΛCDM appear on large scales ($k \sim H_0$) and that they depend on the primordial spectrum of electromagnetic fluctuations. However, the effects on the CMB temperature and matter power spectra are compatible with observations except for very large primordial fluctuations.[12]

§5. Generation of cosmic magnetic fields

It is interesting to note that the ξ-term can be seen, at the equations of motion level, as a conserved current acting as a source of the usual Maxwell field. To see this, we can write $-\xi\nabla^\mu(\nabla_\nu A^\nu) \equiv J^\mu_{\nabla\cdot A}$ which, according to (3·3), satisfies the conservation equation $\nabla_\mu J^\mu_{\nabla\cdot A} = 0$ and we can express (3·2) as:

$$\nabla_\nu F^{\mu\nu} = J^\mu_T \tag{5·1}$$

with $J^\mu_T = J^\mu + J^\mu_{\nabla\cdot A}$ and $\nabla_\mu J^\mu_T = 0$. Physically, this means that, while the new scalar mode can only be excited gravitationally, once it is produced it will generally behave as a source of electromagnetic fields. Therefore, the modified theory is described by ordinary Maxwell equations with an additional "external" current. For an observer

with four-velocity u^μ moving with the cosmic plasma, it is possible to decompose the Faraday tensor in its electric and magnetic parts as: $F_{\mu\nu} = 2E_{[\mu}u_{\nu]} + \frac{\epsilon_{\mu\nu\rho\sigma}}{\sqrt{g}}B^\rho u^\sigma$, where $E^\mu = F^{\mu\nu}u_\nu$ and $B^\mu = \epsilon^{\mu\nu\rho\sigma}/(2\sqrt{g})F_{\rho\sigma}u_\nu$. Due to the infinite conductivity of the plasma, Ohm's law $J^\mu - u^\mu u_\nu J^\nu = \sigma F^{\mu\nu}u_\nu$ implies $E^\mu = 0$. Therefore, in that case the only contribution would come from the magnetic part. Thus, from Maxwell's equations, we get:

$$F^{\mu\nu}{}_{;\nu}u_\mu = \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{g}}B_\rho u_{\sigma\,;\nu}u_\mu = J^\mu_{\nabla\cdot A}u_\mu \tag{5·2}$$

that for comoving observers in a FLRW metric imply (see also[15]):

$$\vec{\omega}\cdot\vec{B} = \rho_g^0 \tag{5·3}$$

where $\vec{v} = d\vec{x}/d\eta$ is the conformal time fluid velocity, $\vec{\omega} = \vec{\nabla}\times\vec{v}$ is the fluid vorticity, ρ_g^0 is the effective charge density today and the $\vec{B}$ components scale as $B_i \propto 1/a$ as can be easily obtained from $\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma;\nu} = 0$ to the lowest order in v. Thus, the presence of the non-vanishing cosmic effective charge density necessarily creates both magnetic field and vorticity, in fact, we find that vorticity grows as $|\vec{\omega}| \propto a$, from radiation era until present.

Notice that $\nabla_\nu A^\nu$ is constant on super-Hubble scales and starts decaying as $1/a$ once the mode reenters the Hubble radius. Thus, today, a mode k will have been suppressed by a factor $a_{in}(k)$ (we are assuming that the scale factor today is $a_0 = 1$). This factor will be given by: $a_{in}(k) = \Omega_M H_0^2/k^2$ for modes entering the Hubble radius in the matter era, i.e. for $k < k_{eq}$ with $k_{eq} \simeq (14\,\mathrm{Mpc})^{-1}\Omega_M h^2$ the value of the mode which entered at matter-radiation equality. For $k > k_{eq}$ we have $a_{in}(k) = \sqrt{2\Omega_M}(1 + z_{eq})^{-1/2}H_0/k$. It is then possible to compute from (4·5) the corresponding power spectrum for the effective electric charge density today $\rho_g^0 = J^0_{\nabla\cdot A} = -\xi\partial_0(\nabla_\nu A^\nu)$. Thus from:

$$\langle\rho(\vec{k})\rho^*(\vec{h})\rangle = (2\pi)^3\delta(\vec{k}-\vec{h})\rho^2(k) \tag{5·4}$$

we define $P_\rho(k) = \frac{k^3}{2\pi^2}\rho^2(k)$, which is given by:

$$P_\rho(k) = \begin{cases} 0, & k < H_0 \\[2ex] \frac{\Omega_M^2 H_0^2 H_{k0}^4}{16\pi^2}\left(\frac{k}{k_0}\right)^{-4\epsilon-2}, & H_0 < k < k_{eq} \\[2ex] \frac{2\Omega_M H_0^2 H_{k0}^4}{16\pi^2(1+z_{eq})}\left(\frac{k}{k_0}\right)^{-4\epsilon}, & k > k_{eq}. \end{cases} \tag{5·5}$$

Therefore the corresponding charge variance will read: $\langle\rho^2\rangle = \int\frac{dk}{k}P_\rho(k)$. Notice that for modes entering the Hubble radius in the radiation era, the power spectrum is nearly scale invariant. Also, due to the constancy of $\nabla_\nu A^\nu$ on super-Hubble scales, the effective charge density power spectrum is negligible on such scales, so that we do not expect magnetic field nor vorticity generation on those scales. Notice that, on
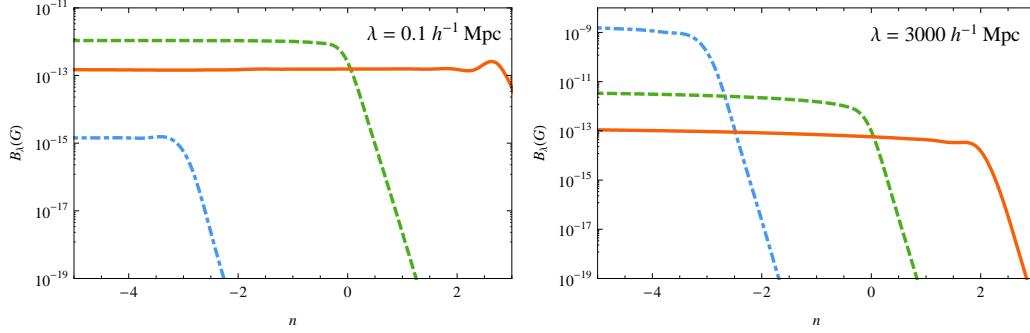
Fig. 2. Lower limits on the magnetic fields generated on galactic scales (left panel) and Hubble horizon scales (right panel) in terms of the magnetic spectral index n for different values of the vorticity spectral index m. Dot-dashed blue for $m = 0$, dashed green for $m \simeq -3$ and full red for $m \simeq -5$.

sub-Hubble scales, the effective charge density generates longitudinal electric fields whose present amplitude would be precisely $E_L \simeq \nabla_\nu A^\nu$.

Using (5·3), it is possible to translate the existing upper limits on vorticity coming from CMB anisotropies[15] into *lower* limits on the amplitude of the magnetic fields generated by this mechanism. We will consider for simplicity magnetic field and vorticity as gaussian stochastic variables such that:

$$\langle B_i(\vec{k}) B_j^*(\vec{h}) \rangle = \frac{(2\pi)^3}{2} P_{ij} \delta(\vec{k} - \vec{h}) B^2(k)$$

$$\langle \omega_i(\vec{k}) \omega_j^*(\vec{h}) \rangle = \frac{(2\pi)^3}{2} P_{ij} \delta(\vec{k} - \vec{h}) \omega^2(k) \qquad (5\cdot6)$$

with $B^2(k) = Bk^n$, $\omega^2(k) = \Omega k^m$ and where $P_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j$ is introduced because of the transversality properties of B_i and ω_i. The spectral indices n and m are in principle arbitrary. In Fig. 2 we show the lower limits on the magnetic fields generated by this mechanism on scales $\lambda = 0.1 h^{-1}$ Mpc, and $\lambda = 3000 h^{-1}$ Mpc, also for inflation at the electroweak scale. We see that fields can be generated with sufficiently large amplitudes in order to seed a galactic dynamo or even to account for observations just by collapse and differential rotation of the protogalactic cloud.[7] Moreover, they could be also compatible with recent extra-galactic observations.[3]

§6. Discussion

In this work, we have shown how a minimal extension of electromagnetism which does not require the introduction of new fields, dimensional parameters or potential terms could provide a simple explanation for the tiny value of the cosmological constant and, at the same time, a mechanism for the generation of magnetic fields on cosmological scales.

Additional aspects of the theory, such as the modification of the magnetohydrodynamical evolution, should be considered in more detail in order to make more precise predictions of the magnetic field amplitudes for different astrophysical objects. Also a complete study of the quantum theory including interactions would

help understanding the physical properties of the new electromagnetic mode. In any case, the results presented in this work provide a clear example of the potential cosmological implications which could appear in the interplay between gravity and quantum gauge theories.

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